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USE OF CHARTS FOR FLOW DISCHARGE CALCULATIONS

By O. Lutz

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 972

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SUMMARY

Various problems in connection with engine design
involve flow-discharge calculations which are rendered
difficult both on account of the large number of external
variables that enter into the computation - i.e., changes
in discharge area during the process, change in volume of
the cylinder, pressure, etc., and changes in the thermal
constants themselves of the flow medium. A fairly accu-
rate solution that does not involve an excessive amount
of labor can be obtained only through the extensive use
of i-s tables. In the present report, a·solution is of-
fered in the form of a different method making use of the
I-S tables of Lutz and Wolf.

1. INTRODUCTION

Those flow processes of most interest in connection
with engine design occur mainly in the supercritical
region at temperatures for which the specific heats, and
also the adiabatic coefficient, must no longer be con-
sidered as constant. For these conditions i-s charts are
employed. The simplest and most convenient for this pur-
pose are the I-S charts of O. Lutz and F. Wolf. On ac-
count of the large number of external variables involved -
discharge area, cylinder volume, back pressure, etc. - a
straightforward numerical solution of the problem is im-
possible and attempts in this direction lead only to com-
plications (reference 1).

A different method is here developed which has the
advantage of clearness and which takes into account all
the variables that enter the problem.

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*Die graphische Ermittlung von Ausströmvorgängen. Luft-
fahrtforschung, vol. 17, no. 8, Oct. 26, 1940, pp.
332-335.

2. DERIVATION OF THE RELATIONS USED

The discharge is assumed to take place from a working cylinder (fig. 1) whose volume at any instant is determined by the piston area f_k and the displacement h of the piston from the cylinder head. With curved cylinder head or piston head, the value of h is given by the condition that the ratio of h_{max} to the piston stroke h_o is equal to the ratio of ϵ to $\epsilon - 1$ where ϵ is the compression ratio. The discharge area (taking into account also constrictions in the jet) will be denoted by f, the discharge velocity at the minimum jet cross section by w_a. Magnitudes characterizing the state of the gas in the cylinder will have the subscript 1.

For w_a, we have

$$w_a = 91.5 \sqrt{\frac{I_1 - I}{M}} \qquad (1)$$

where I is the internal heat or enthalpy in the discharge area and M the gram molecular weight of the discharge medium. In the I-S tables referred to, (page 7), the method is indicated of obtaining these values for combustion gases whose thermal properties deviate from those of air. The internal heat I in the discharge section will be equal to that in the external region only for the subcritical flow condition; for the supercritical condition it must be separately determined.

Assuming the process is adiabatic, we have for the change in state in the cylinder

$$\frac{dp_1}{p_1} = \frac{C_{p_1}}{C_{v_1}} \frac{dG_1}{G_1} - \frac{C_{p_1}}{C_{v_1}} \frac{dh}{h} \qquad (2)$$

where C denotes the gram molecular heat, G_1 the instantaneous weight of the gas and dh/h the relative change in volume. The time rates of change are connected by

$$\frac{dp_1}{dt} = \frac{C_{p_1}}{C_{v_1}} \frac{p_1}{\gamma_1 f_k h} \frac{dG_1}{dt} - \frac{C_{p_1}}{C_{v_1}} \frac{p_1}{h} \frac{dh}{dt};$$

Now dh/dt is the instantaneous piston velocity c and dG_1/dt may be replaced by $-\gamma f w_a$ so that

$$\frac{dp_1}{dt} = -\frac{\underline{C}_{p_1}}{\underline{C}_{v_1}} \frac{p_1}{h} \left(\frac{f}{f_k}\,\frac{\gamma}{\gamma_1}\,w_a + c\right)$$

which may also be written

$$\frac{dp_1}{dt} = -\frac{\underline{C}_{p_1}}{\underline{C}_{v_1}} \frac{p_1}{h} \left(\frac{f}{f_k}\,\frac{\underline{V}_1}{\underline{V}}\,w_a + c\right) \tag{3}$$

where $\underline{V}$ is the gram molecular volume.

It is convenient in engine computations to refer to the crank angle φ. Since the time element dt and the crank-angle element $d\varphi$ are connected by the relation

$$dt = \frac{d\varphi}{6n} \qquad (n = rpm)$$

we have finally, passing to finite differences Δp_1

$$\Delta p_1 = -\frac{\underline{C}_{p_1}}{\underline{C}_{v_1}} \frac{p_1}{6nh} \left(\frac{f}{f_k}\,\frac{\underline{V}_1}{\underline{V}}\,w_a + c\right) \Delta\varphi \tag{3a}$$

The known external variables are the revolutions per minute n, the distance of the piston from the cylinder head h, the ratio of cross sections f/f_k, and the instantaneous piston velocity c; the initial state p_1, $\underline{V}_1$ must furthermore be given. The ratio $\kappa_1 = \underline{C}_{p_1}/\underline{C}_{v_1}$ of the specific heats and the ratio $\underline{V}_1/\underline{V}$ of the gram molecular volumes for supercritical flows must be determined by further considerations.

In figure 2, the values of κ are plotted as functions of the temperature with the factor β of the I-S tables (page 5) as parameter. It may be pointed out in this connection that the values of κ can also be

directly determined from the I-S tables if not too great
accuracy is required. For the constant-pressure curves
of the I-S tables we have, as is known,

$$d\underline{S}_p = \underline{C}_p \, \frac{dT}{T}$$

and for the constant-volume curves

$$d\underline{S}_v = \underline{C}_v \, \frac{dT}{T}$$

Again passing to finite differences, we obtain for the re-
quired ratio of the specific heats

$$\frac{\underline{C}_p}{\underline{C}_v} = \frac{\dfrac{\Delta \underline{S}_p}{\Delta T}}{\dfrac{\Delta \underline{S}_v}{\Delta T}} \qquad\qquad (4)$$

Accordingly, to find $\underline{C}_p/\underline{C}_v$, the increments $\Delta \underline{S}_v$ and
$\Delta \underline{S}_p$ between the isotropes, isobars and isochores (con-
stant-volume curves) corresponding to a small increment
ΔT from the state a, are read off on the I-S chart as
indicated in figure 3. We then have

$$\frac{\underline{C}_p}{\underline{C}_v} = \frac{\Delta \underline{S}_p}{\Delta \underline{S}_v} \qquad\qquad (4a)$$

To reduce the error due to the curvature of the curves,
it is best to carry out the process for positive and
negative temperature differences and average the results.

The exact determination of the critical pressure
ratio from which the discharge velocity for supercritical
flows and the corresponding molecular volume are obtained
is not possible in explicit form. Assuming that, for
supercritical flows, the weight passing through the dis-
charge area $f \dfrac{w_a}{\underline{V}_s}$ remains a maximum, we must have

$$\frac{dw_a}{w_a} - \frac{dV_s}{V_s} = 0 \qquad (5)$$

or using (1)

$$- \frac{dI_s}{2(I_1 - I_s)} = \frac{dV_s}{V_s}$$

Now $dI_s = C_{p_s} dT_s$ and $\dfrac{dV_s}{V_s}$ for an adiabatic process

$= - \dfrac{C_{v_s}}{AR} \dfrac{dT_s}{T_s}$ so that we have finally

$$\frac{I_1 - I_s}{T_s} = \frac{C_{p_s}}{2}\left(\frac{C_{p_s}}{C_{v_s}} - 1\right) \qquad (6)$$

This equation, which as yet has not been simplified by neglecting terms, can be solved only graphically. The temperature ratio T_s/T_1 was determined from the I-S chart, number 1, I_1 being determined from the state I_s, T_s.

Here, too, an approximate method can be indicated. Equation (6) can be simplified by assuming the specific heat for the expansion process from the inside of the cylinder to the opening to be constant and equal to the value C_{p_s} in the opening. We then have

$$\frac{T_1}{T_s} = \frac{C_{p_s} + C_{v_s}}{2C_{v_s}} \qquad (6a)$$

The right-hand side is determined as in figure 3 except that the segment II is taken only up to the midpoints between the lines of constant volume and the isobars, as shown in figure 4. We then have

$$\frac{T_s}{T_1} = \frac{I}{II} \qquad (6b)$$

Thus T_s and therefore the state in the I-S chart can be determined.

For the case of flows below the critical condition, the final state is that of the external region and hence known.

All the magnitudes in formula (3a) are now known and the discharge can be computed step by step without too much labor.

3.   ILLUSTRATIVE EXAMPLE

The discharge process from a cylinder of 1.64 liters volume will be determined, the initial state being given by $p_1 = 6.0$ atm, $\vartheta_1 = 1400°$; the valve opening time is shown in figure 5. According to equation (3a) for the determination of the pressure variation, there are first required to be known the values h, c and f/f_k associated with the engine dimensions. The variation of these values with the crank angle is shown in figure 5. Since f/f_k denotes the ratio of the minimum jet cross section (not the valve cross section f_v) to the piston area, it is necessary by a separate test to determine the contraction coefficient ψ of the valve under consideration. In figure 6, the values of ψ for a few valve arrangements are plotted.* Corresponding to these data, curve 4 has been chosen as approximately applying to our case and f/f_k determined in figure 5.

It was found convenient to use steps of $10°$ crank angle for the supercritical discharge region and $5°$ for the subcritical.

For the first step, the initial values are $p_1 = 6$ atm, $\vartheta_1 = 1400°$, $h_{55°} = 0.141$ m, $c_{55°} = 15.63$ m/s and $\left(\dfrac{f}{f_k}\right)_{55°}$ $= 0.00083$. The ratio $\dfrac{c_{p_1}}{c_{v_1}}$ of the specific heats at the

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*According to tests conducted by E. Hummel at the Institut für Motorenforschung der Luftfahrtforschungsanstalt Hermann Görung. In applying statically measured values to dynamic processes, it should be remembered that the value correction factors may vary as a result of possible pulsations in discharging. Account will be taken of this fact in a later report.

initial temperature $T_1 = 1673^O$ K is found from figure 2 to have the value 1.262 (where, for the combustion gas, a stoichiometric gasoline-air mixture of table factor β = 1.50 is to be assumed, see I-S charts). From figure 2, the temperature ratio T_S/T_1 is obtained as 0.882 with the aid of which the molecular volume ratio $\underline{V}_1/\underline{V}$ is found from the I-S charts to have the value 0.612. The discharge velocity w_a is finally obtained as 743 m/s. We thus have for the first step:

$$\Delta p = -\ 1.262\ \frac{6.0}{6 \times 2700 \times 0.141}\ (0.00083 \times 0.612 \times 743 + 15.63) \times 10$$

$$= -\ 0.530\ \text{atm}$$

This pressure drop is large so that the first step (for which the initial values of 6.0 atm and 1400° were used and not the average values which should have been substituted for more accurate computation) must be corrected. The computation is therefore repeated with $p_1 = 5.74$ atm and $\vartheta_1 = 1384^O$ (from the I-S charts) and there is obtained

$$\Delta p = -\ 1.262\ \frac{5.740}{6 \times 2700 \times 0.141}\ (0.00083 \times 0.620 \times 783 + 15.63) \times 10$$

$$= -\ 0.508\ \text{atm}$$

There is then obtained for the next step the initial value $p_{150}0 = 5.492$ atm and from the I-S charts $\vartheta_{150}0 = 1369^O$. These values are indicated in figure 7. It is now possible with sufficient accuracy to estimate the mean computation value $p_{145}0$ for the second step. Generally no correction will be required.

The accuracy of the computation procedure depends on

a) the size of the computation steps,

b) the accuracy with which the values $\dfrac{c_{p_1}}{c_{v_1}}$, p_1, $\dfrac{\underline{v}_1}{\underline{v}}$,

and w_a (see equation (3a)) can be estimated or corrected before each half step,

c) the accuracy in reading off the I-S charts in
   determining values ϑ_1 and w_a. It is better
   in place of V to obtain the corresponding
   values p and ϑ which can be more accurate-
   ly read off and then determine V from the
   gas equation.

Figure 7 shows the results of the computation. On
figure 8 is shown finally the weight of gas G* dis-
charged per unit of time. The curve has a maximum short-
ly after reaching inner dead center and toward the end of
the process has a slight irregularity which is due to the
large piston velocity.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

REFERENCE

1. Schüle, W.: Technische Thermodynamik. Bd. 2, S. 390,
       Berlin, 1923.
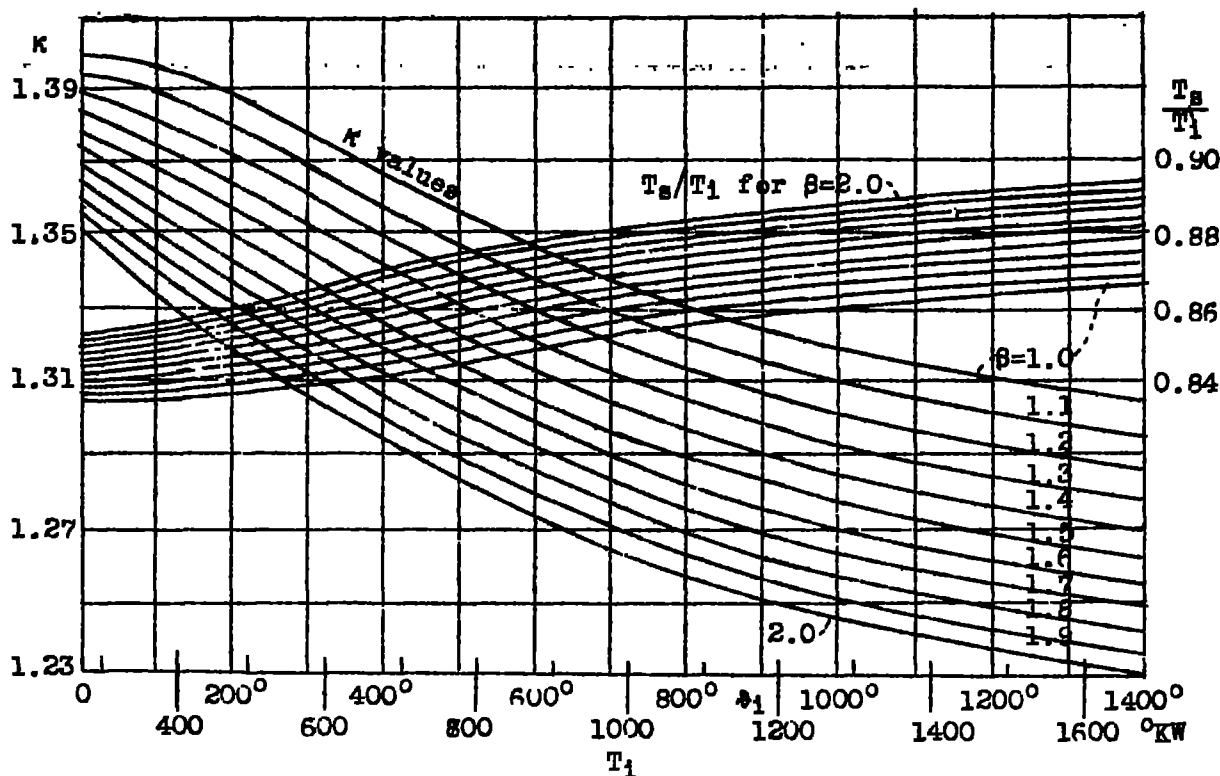
Figure 2.- Values of κ and $\dfrac{T_s}{T_1}$ as functions of the temperature and the chart factor β .
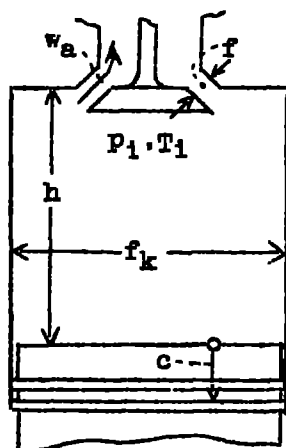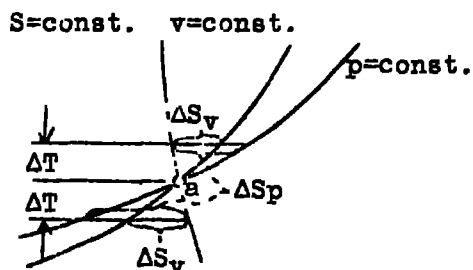

Figure 1.- Sketch indicating notation.

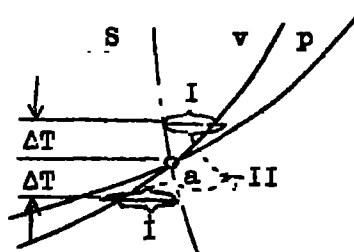Figure 3.- Determination of κ.

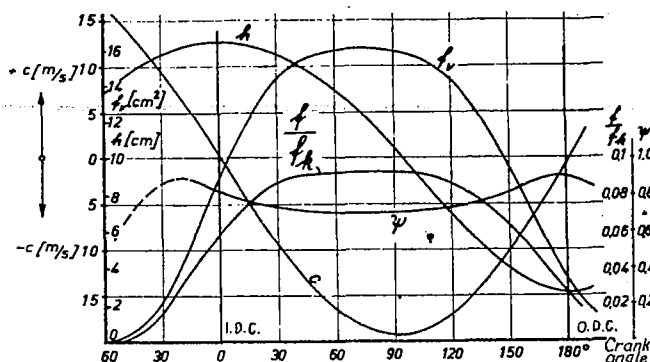

Figure 4.- Determination of $\dfrac{T_s}{T_1}$

Figure 5.— Piston displacement h, piston velocity c, valve cross section f_v , contraction coefficient ψ and ratio f/f_k of the discharge area to the piston area.

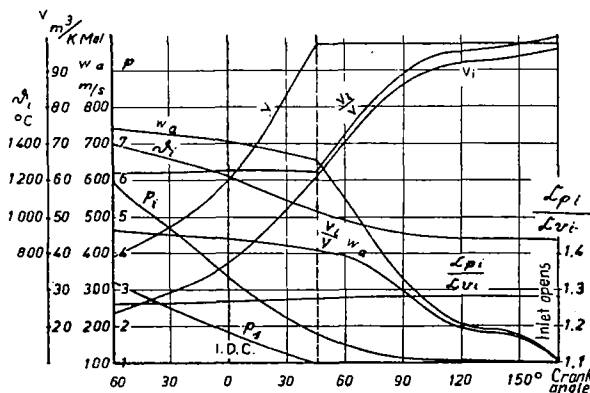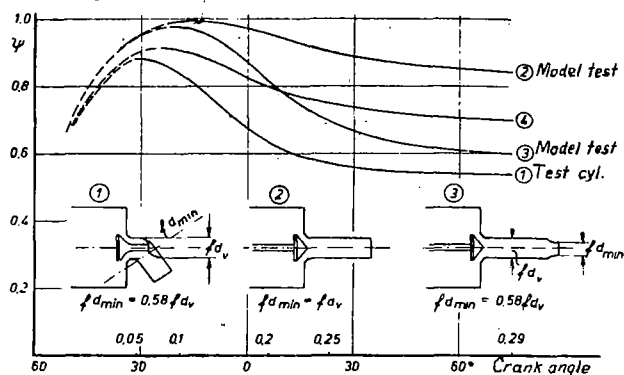Figure 6.— Contraction coefficient ψ for various valve arrangements.

Figure 7.— Graphical determination of the discharge process.

Figure 8.— Discharge weight G* and valve lift h_v as a function of the crank angle position.